

APPENDIX A -- TIMBER MODELS

This appendix explores how the three different timber models (baseline, dieback, and regeneration) are formulated. In order to understand the interesting economic effects that occur in the climate scenarios, first order conditions are derived for the choice variables in the models. The models are solved using GAMS and the MINOS5 solver.

BASELINE GLOBAL TIMBER MARKET MODEL

Each hectare of timberland supplies merchantable timber logs according to the following yield function:

$$(A.1) \quad V_i(a_i(t); m_i(t_0)) = [\phi_i(m_i(t_0) + 1)^{\eta_i}] V_i(a_i).$$

Where

$$(A.2) \quad V_i(a_i) = \exp(\alpha_i - \beta_i/a_i).$$

The letter i indicates different timber types, $a_i(t)$ is the age of timber, $m_i(t_0)$ is the quantity of management intensity (≥ 0), ϕ_i is the natural stocking density, η_i is a parameter representing the returns to additional management, and α_i and β_i are estimated yield function parameters. Estimates of parameters are derived from Sohngen (1996) and Sedjo and Lyon (1990).

Society consumes these products according to the inverse demand function in equation (1) in the text. Net market surplus is consumer surplus minus the costs of producing timber products, $C_H(Q(t))$. One set of costs are those associated with harvesting and transporting timber to markets, given as $c_i^H(q_i(t))$ for region i . A second set of costs are those associated with accessing inaccessible stocks, given as $c_i^A(q_i(t))$ for region i . The third set of costs are those associated with regeneration of stocks. The costs of regenerating stocks that have previously been harvested are given as $C_G(t)$. Because there may be additional costs of regenerating forests on land that has previously been in agriculture or some other use, we assume that $C_N(t)$ represents the costs of converting land to plantations. The final set of costs are land rental costs. These costs are given as $R_i(X_i(t))$ for each timber type. Given these conditions, net surplus is written as

$$(A.3) \quad S(\cdot) = \int_0^{Q^*(t)} \{D(Q(t), Z(t)) - C_H(Q(t))\} dQ(t) - C_G(t) - C_N(t) - \sum_i R_i(X_i(t)),$$

Global timber markets are assumed to maximize the net present value of net market surplus,

$$(A.4) \quad \underset{H_i(t), G_i(t), N_i(t), m_i(t)}{Max} \int_0^{\infty} \{S(\cdot)\} e^{-rt} dt$$

subject to:

$$(A.5) \quad \dot{X}_i = -H_i(t) + G_i(t) + N_i(t), \quad \forall i$$

$$(A.6) \quad X_i(0) = X_{i,0} \quad \forall i$$

$$(A.7) \quad X_i(t), H_i(t), G_i(t), N_i(t), m_i(t) \geq 0 \quad \forall i$$

This problem can be analyzed by forming a Hamiltonian,

$$(A.8) \quad \dot{h} = S(H(t), Z(t), G(t), X(t)) + \sum_i \mu_i(t) [-H_i(t) + G_i(t) + N_i(t)].$$

Solving the Hamiltonian reveals the following first order conditions

$$(A.9) \quad S_{H_i}(\cdot) = \mu_i(t) \quad \forall i$$

$$(A.10) \quad S_{G_i}(\cdot) = -\mu_i(t) \quad \forall i$$

$$(A.11) \quad S_{m_i}(\cdot) = 0 \quad \forall i$$

$$(A.12) \quad S_{N_i}(\cdot) = -\mu_i(t) \quad \forall i$$

$$(A.13) \quad \dot{\mu}_i - r\mu_i(t) = R_i(t) \quad \forall i$$

$$(A.14) \quad \dot{X}_i = -H_i(t) + G_i(t) + N_i(t) \quad \forall i$$

We first consider the conditions under which timber stands will be harvested.

Equations (A.9) and (A.11) can be combined to obtain (note that $\forall i$ has been suppressed below):

$$(A.15) \quad \dot{S}_{H_i} = rS_{H_i} + R_i(t).$$

The left hand side of (A.15) is the marginal benefit of waiting to harvest a timber stand until the next period, and the right hand side is the marginal opportunity cost of not harvesting the stand and holding the land one more period. The first order conditions described in the text are determined by assuming a linear demand function for $D(Q(t), Z(t))$, constant marginal harvesting costs equal to c_i^H , and an upward sloping access cost function for each inaccessible forest type given as $c_i^A(H_i(t)V_i(a_i(t); m_i(t_0)))$. Note that $q_i(t)$ is the area harvested times the yield, or $H_i(t)V_i(a_i(t); m_i(t_0))$. Units of regeneration intensity, $m_i(t_0)$ can be purchased at a constant price, $p_{m,i}$. Under these conditions, $S(\cdot)$ becomes

$$(A.16)$$

$$S(\cdot) = k + A \left[\sum_i H_i(t)V_i(a_i(t); m_i(t_0)) \right] - B \left[\sum_i H_i(t)V_i(a_i(t); m_i(t_0)) \right]^2 - \sum_i c_i^H H_i(t)V_i(a_i(t); m_i(t_0)) - \sum_{i \in inac} c_i^A(H_i(t)V_i(a_i(t); m_i(t_0))) - \sum_i p_m m_i(t)G_i(t) - \sum_{i \in emerg} f_{N,i}[N_i(t)] - \sum_i R_i(t)X_i(t)$$

In equation (A.16), $i \in inac$ indicates that the species are part of the inaccessible stocks of timber to note that access costs apply only to those stocks, and $i \in emerg$ indicates that

the species are part of the emerging region plantation stocks to note that additional regeneration costs apply only to those stocks. With (A.16), we solve for S_H for alternative timber stocks. For accessible timber stocks this is

$$(A.17) S_{H_i} = [A - 2BH_i(t)V_i(a_i(t); m_i(t_0))]V_i(a_i(t); m_i(t_0)) - c_i^{H'}V_i(a_i(t); m_i(t_0))$$

In a competitive market, $[A - 2BH_i(t)V_i(a_i(t); m_i(t_0))]$ is the global price of timber logs, $P(t)$, and $c_i^{H'}$ is the marginal cost of harvesting and transporting those logs to markets.

Thus,

$$(A.18) S_{H_i} = (P(t) - c_i^{H'})V_i(a_i(t); m_i(t_0)).$$

Equation (11) in the text is obtained by taking the time derivative of (A.18) and substituting that result into (A.15), assuming that harvesting and transporting costs are constant over time. This is

$$(A.19) \dot{P}V_i(a_i(t); m_i(t_0)) + (P(t) - c_i^{H'})\dot{V}_i = r(P(t) - c_i^{H'})V_i(a_i(t); m_i(t_0)) + R_i(t)$$

For inaccessible timberlands, where access costs are an important component of the total value of timber stands, note that S_H for inaccessible timberlands will be

$$(A.20) S_{H_i} = (P(t) - c_i^{H'} - c_i^{A'})V_i(a_i(t); m_i(t_0)),$$

where $c_i^{A_1}$ are marginal access costs. Since these stocks are essentially old growth, annual timber growth is 0, and land rental rates will be also be 0 since this land is fairly unproductive and there are few alternative uses. Equation (A.19) will be altered under these conditions to reveal equation (12) in the text.

Turning to regeneration intensity, S_G can be determined from (A.16) as

$$(A.21) \quad S_{G_i} = -p_m m_i(t_0).$$

To determine the appropriate first order condition for this, note that $\mu_i(t)$ is the marginal net surplus of holding one additional hectare of land in timber until the next regeneration decision at time t_f :

$$(A.22) \quad \mu_i(t_0) = (P(t_f) - c_i^{H_1})V_i(t_f - t_0; m_i(t_0))e^{-r(t_f - t_0)} - \int_{t_0}^{t_f} \{R_i(s_i)e^{-rs_i}\} ds_i$$

Substituting (A.21) and (A.22) into (A.10) reveals

$$(A.23) \quad (P(t_f) - c_i^{H_1})V_i(t_f - t_0; m_i(t_0))e^{-r(t_f - t_0)} = p_m m_i(t_0) + \int_{t_0}^{t_f} \{R_i(s_i)e^{-rs_i}\} ds_i,$$

which is equation (13) in the text.

While equation (A.23) can be used to determine the area of timberland regenerated, timberland owners also have a choice over the intensity with which they manage timber

stocks. To see how these decisions are made, note that S_m is the marginal net surplus of an additional unit of regeneration intensity at time t_0 over $G_i(t_0)$ hectares of land:

$$(A.24) \quad S_{m_i} = (P(t_f) - c_i^H) \left(\frac{dV_i(t_f - t_0; m_i(t_0))}{dm_i(t_0)} \right) \left(e^{-r(t_f - t_0)} \right) G_i(t_0) - p_{m,i} G_i(t_0) = 0$$

Increasing management on stocks planted today has an effect on future timber yield through $m_i(t_0)$. Landowners continue adding management inputs until the net present value of future marginal benefits equal the present marginal costs.

The decision to establish new timber plantations in the subtropical emerging region in equation (15) is found by first solving S_N , for $i \in \text{emerg}$.

$$(A.25) \quad S_{N_i}(\cdot) = p_m m_i(t_0) + f_{N,i}'[N_i(t_0)].$$

The marginal net surplus of an additional hectare of land in plantations held until the next regeneration decision, $\mu_i(t_0)$, is

$$(A.26) \quad \mu_i(t_0) = (P(t_f) - c_i^H) V_i(t_f - t_0; m_i(t_0)) e^{-r(t_f - t_0)} - \int_{t_0}^{t_f} \{ R_i(s_i) e^{-rs_i} \} ds_i .$$

Combining (A.12) with (A.25) and (A.26) reveals

$$(A.27) \quad (P(t_f) - c_i^{H'}) V_i(t_f - t_0; m_i(t_0)) e^{-r(t_f - t_0)} = p_{m,i} m_i(t_0) + f_{N,i}'(N_i(t_0)) + \int_{t_0}^{t_f} [R_i(s_i) e^{-rs_i}] ds_i$$

This equation shows that landowners invest in additional hectares of timber plantations until the net present value of future benefits equals the net present value of costs.

THE DIEBACK SCENARIO

The timber model is altered in three ways in the dieback scenario. First, the stock of timber is affected directly by dieback according to $\delta_i(\kappa(t))$. This stimulus will alter equation (A.5) to

$$(A.28) \quad \dot{X}_i = -H_i(t) - \delta_i(\kappa(t)) X_i(t) + G_i(t) + N_i(t)$$

Second, the quantity harvested will be altered because some stock is entering markets strictly due to dieback. The proportion of salvage in each timber type is γ_i , which varies from 0 to 75%, depending on access and value. Finally, in addition to the direct effects of dieback, stocks will slowly adjust to changing growing conditions as timber growth responds to climate change, as captured by $\theta_i(\kappa(t))$. $S(\cdot)$ is rewritten as

$$(A.29)$$

$$\begin{aligned}
S(\cdot) = & k + A \left[\sum_i H_i(t) V_i(a_i(t); m_i(t_0), \theta_i(\kappa(t))) + \sum_i \delta_i(\kappa(t)) \gamma_i X_i(t) V_i(a_i(t); m_i(t_0), \theta_i(\kappa(t))) \right] - \\
& B \left[\sum_i H_i(t) V_i(a_i(t); m_i(t_0), \theta_i(\kappa(t))) + \sum_i \delta_i(\kappa(t)) \gamma_i X_i(t) V_i(a_i(t); m_i(t_0), \theta_i(\kappa(t))) \right]^2 - \\
& \sum_i c_i^{H'} (H_i(t) V_i(a_i(t); m_i(t_0), \theta_i(\kappa(t))) + \delta_i(\kappa(t)) \gamma_i X_i(t) V_i(a_i(t); m_i(t_0), \theta_i(\kappa(t)))) - \\
& \sum_{i \in \text{inac}} c_i^A [H_i(t) V_i(a_i(t); m_i(t_0), \theta_i(\kappa(t))) + \delta_i(\kappa(t)) \gamma_i X_i(t) V_i(a_i(t); m_i(t_0), \theta_i(\kappa(t)))] - \\
& \sum_i P_m m_i(t) G_i(t) - \sum_{i \in \text{emerg}} f_{N,i} [N_i(t)] - \sum_i R_i(t; \kappa(t)) X_i(t)
\end{aligned}$$

With this function, a new hamiltonian can be formulated and solved. The first order condition on harvesting accessible timber stocks during dieback will become

$$\begin{aligned}
\text{(A.30) } \dot{P} V_i(a_i(t); m_i(t_0), \theta_i(\kappa(t))) + (P(t) - c_i^{H'}) \left(\frac{dV_i}{d\theta_i} \frac{d\theta_i}{dt} \right) = \\
[r + (1 - \gamma_i) \delta_i(\kappa(t))] (P(t) - c_i^{H'}) V_i(a_i(t); m_i(t_0), \theta_i(\kappa(t))) + R_i(t; \kappa(t))
\end{aligned}$$

Climate change alters the marginal harvest conditions, and the time path of harvest in each region. For example, climate change may increase or decrease timber growth through the ecological effects of $\theta_i(t)$. The effect of both annual growth and the effect of climate change on the annual growth is measured with $\left(\frac{dV(\cdot)}{d\theta_i} \frac{d\theta_i}{dt} \right)$. The effect of the ecological stimulus on the marginal benefits will depend on both the size and direction of the ecological impacts, as well as global prices. Because dieback will increase the marginal cost of waiting to harvest a stand, harvests will increase in forests that are most

susceptible to dieback. The optimal adjustment marginally shifts timberland towards younger age classes so that stands with less timber, on average, are dying back.

Climate change can similarly influence harvest activity in inaccessible forests. The first order condition in equation (12) in the text becomes

$$(A.31) \quad \dot{P}V_i(a_i(t); m_i(t_0), \theta_i(\kappa(t))) + (P(t) - c_i^H - c_i^A) \left(\frac{dV_i}{d\theta_i} \frac{d\theta_i}{dt} \right) = \\ [r + \delta_i(\kappa(t))] (P(t) - c_i^H - c_i^A) V_i(a_i(t); m_i(t_0), \theta_i(\kappa(t))) + R_i(t; \kappa(t))$$

The marginal benefits of waiting to harvest, the left hand side, are affected by changes in the growth rate of timber. Because climate change may induce growth in this region if forests become more productive, it may be optimal to harvest some forests in later periods when these growth effects have a large influence. The marginal costs of waiting will also be affected by climate change, although we assume that γ_i is 0 in inaccessible stocks (most of the dieback occurring in this region is distant from access roads and markets). Marginal costs are affected both by dieback and land rental rates, $R_i(t; \kappa(t))$. If inaccessible land becomes much more productive during climate change, rental rates may increase (depending on price effects), thereby increasing the marginal costs of waiting to harvest, and increasing harvests relative to the baseline case. These effects depend on the predicted change in production in these regions, but inaccessible forests may provide important sources of timber to markets during climate change.

When landowners face the possibility of future dieback, it will affect their decisions to regenerate because dieback alters the expected future yield of timber. To see this, we note that in any given year, the expected yield of timber for stands undergoing dieback is

$$(A.32) \quad E[V_i(a_i(t); m_i(t_0), \theta_t(\kappa(t)))] = \\ (1 - \delta_i(\kappa(t)))V_i(a_i(t); m_i(t_0), \theta_t(\kappa(t))) + \delta_i(\kappa(t))\gamma V_i(a_i(t); m_i(t_0), \theta_t(\kappa(t)))$$

Over the time the stand is susceptible to dieback this is

$$(A.33) \quad E[V_i(a_i(t); m_i(t_0), \theta_t(\kappa(t)))] = V_i(a_i(t); m_i(t_0), \theta_t(\kappa(t))) \prod_{t_0}^{t_f} (1 - (1 - \gamma_i)\delta_i(\kappa(t))).$$

We assume that the future proportion (but not the exact stocks) of timber that dies back, $\delta_i(\kappa(t))$, is known at time t_0 . This means that the expected yield, $E[V(\cdot)]$, can be determined in each time period when a forester must make regeneration decisions. Note that we use a certainty equivalent approach, not a stochastic approach, to model dieback. When the regeneration decision occurs, the marginal net surplus of an additional hectare of land in timber species that are undergoing dieback is:

$$(A.34) \quad \mu_i(t_0) = (P(t_f) - c_i^{H'})E[V(a_i(t_f); m_i(t_0), \theta_t(\kappa(t)))]e^{-r(t_f-t_0)} - \int_{t_0}^{t_f} R_i(s; \kappa(s))e^{-rs} ds$$

Landowners continue to regenerate stocks until the marginal benefits of regenerating one more hectare of land in timber equal the marginal costs

(A.35)

$$(P(t_f) - c_i^{H'}) E[V(a_i(t_f); m_i(t_0), \theta_i(\kappa(t)))] e^{-r(t_f - t_0)} = p_{m,i} m_i(t_0) + \int_{t_0}^{t_f} R_i(s; \kappa(s)) e^{-rs} ds$$

The effects of dieback reduce the potential yield below what might be expected without climate change, so the marginal benefits are lower than without dieback. This will reduce the incentive to regenerate stocks that face heavy dieback effects.

The decision of how intensively to manage the regeneration effort depends similarly on future yield and potential dieback. Stocks that are not susceptible to dieback will be regenerated as in (A.24) above. Substituting the yield function in equation (A.33) into equation (A.24) shows how regeneration decision are affected in stocks that are undergoing dieback. Additional management intensity will be purchased up to the point where the net present value of the expected future marginal benefits equal the cost of additional management today,

$$(A.36) \quad (P(t_f) - c_i^{H'}) \left(\frac{dE[V_i(a(t_f); m_i(t_0), \theta_i(\kappa(t)))]}{dm_i(t_0)} \right) e^{-r(t_f - t_0)} = p_{m,i}$$

Dieback reduces the discounted future benefits of management, so that landowners will spend fewer resources regenerating stocks that are susceptible to dieback.

The decision to expand timber plantations depends entirely on future growth effects. We assume that managers only expand in regions where dieback is not a factor. The marginal regeneration decision for plantations can be expressed as:

(A.37)

$$(P(t_f) - c_i^H) V_i(t_f - t_0; m_i(t_0), \theta_i(k(t))) e^{-r(t_f - t_0)} = p_{m,i} m_i(t_0) + f_{N,i}'(N_i(t_0)) + \int_{t_0}^{t_f} [R_i(s_i, \kappa(t)) e^{-rs_i}] ds_i$$

As elsewhere, the decision to establish additional plantations depends on future timber yields. If timber yields are increasing relative to the baseline, this increases the marginal benefits on the left hand side, and it potentially increases the area of land in plantations.

THE REGENERATION SCENARIO

The second stock redistribution scenario is "regeneration." Although land shifts from one timber type to another, land only shifts at the time of regeneration. Existing timber stocks are not destroyed during climate change. When climate changes enough for a given piece of land to shift from one timber type to another, the existing stand continues growing until it is harvested or dies of natural senescence. The conversion of timberland from one timber type to another during climate change therefore depends on determining the economically optimal age at which to make this conversion.

Stocks that shift from one timber type to another during climate change are separated from those that remain in their original timber type. These stocks denoted as "e," and stocks that remain in their initial timber type are denoted "i," as before. The equation of

motion in (A.14) remains the same, but includes two sets of stocks, I for stocks not undergoing change and E for stocks undergoing change. Net surplus becomes

(A.38)

$$\begin{aligned}
S(\cdot) = & k + A \left[\sum_{i \in I} H_i(t) V_i(a_i(t); m_i(t_0), \theta_i(\kappa(t))) + \sum_{i \in E} H_i(t) V_i(a_i(t); m_i(t_0), \theta_i(\kappa(t))) \right] - \\
& B \left[\sum_{i \in I} H_i(t) V_i(a_i(t); m_i(t_0), \theta_i(\kappa(t))) + \sum_{i \in E} H_i(t) V_i(a_i(t); m_i(t_0), \theta_i(\kappa(t))) \right]^2 - \\
& \sum_{i \in I} c_i^H (H_i(t) V_i(a_i(t); m_i(t_0), \theta_i(\kappa(t)))) - \sum_{i \in E} c_i^H (H_i(t) V_i(a_i(t); m_i(t_0), \theta_i(\kappa(t)))) - \\
& \sum_{i \in I \in \text{inac}} c_i^A ((H_i(t) V_i(a_i(t); m_i(t_0), \theta_i(\kappa(t)))) - \sum_{i \in E \in \text{inac}} c_i^A ((H_i(t) V_i(a_i(t); m_i(t_0), \theta_i(\kappa(t)))))) \\
& \sum_{i \in I} p_m m_i(t) G_i(t) - \sum_{i \in E} p_m m_i(t) G_i(t) - \sum_{i \in \text{emerg}} f_{N,i} [N_i(t)] N_i(t) - \sum_{i \in I} R_i(t) X_i(t) - \sum_{i \in E} R_i(t) X_i(t)
\end{aligned}$$

With (A.38), the hamiltonian can be formulated and solved. The decision to harvest timber stocks in both sets I and E are affected only by the slowly shifting yield functions through $\theta_i(\kappa(t))$:

(A.39)

$$\begin{aligned}
\dot{P} V_i(a_i(t); m_i(t_0), \theta_i(\kappa(t))) + (P(t) - c_i^H) \left(\frac{dV_i}{d\theta_i} \frac{d\theta_i}{dt} \right) = \\
rP(t) V(a_i(t); m_i(t_0), \theta_i(\kappa(t))) + R(t; \kappa(t))
\end{aligned}$$

For stocks in the set I (where the only influence of climate change is to alter future growth rates), harvests are altered only by the affects of changing growth rates on the marginal benefits and marginal costs. While these same conditions hold for stocks in set

E, the marginal costs will reflect future rental values based on the new timber type and yield function existing on the land. If the new type is more (less) productive than the old type, then the marginal costs of waiting to harvest this timber type will increase (decrease) relative to the baseline, and stocks will be harvested more rapidly (slowly).

The harvest conditions for inaccessible timber stands will also change under the regeneration scenario

$$(A.40) \quad \dot{P}V_i(a_i(t); \theta_i(\kappa(t))) + (P(t) - c_i^H - c_i^A) \left(\frac{dV(\cdot)}{d\theta_i} \frac{d\theta}{dt} \right)_i =$$

$$r(P(t) - c_i^H - c_i^A) V_i(a_i(t); \theta_i(\kappa(t))) + R_i(t; \kappa(t))$$

In the regeneration scenario, harvests of inaccessible stocks depend on changes in growth rates and land rent. Climate change may induce net timber growth or declines due to the ecological effects of $\theta_i(t)$. If climate change enhances growing conditions in these regions, there are two effects. First, timberland rental values may become positive and increase the marginal costs of waiting to harvest these stocks. This would tend to increase harvests relative to the baseline. If yields are increasing in this region however, the marginal benefits of waiting to harvest will also increase. It is not clear *a priori* what the net effect of climate change on the area of inaccessible stocks harvested will be. It will depend entirely on the entire time path of ecological and economic changes that occur.

Regeneration decisions in stocks that do not shift from one timber type to another will be influenced only by future changes in growth $\theta_i(\kappa(t))$. Landowners will continue regenerating until the present value of future marginal benefits equal the marginal costs,

(A.41)

$$(P(t_f) - c_i^{H,i}) V_i(a_i(t); m_i(t_0), \theta_i(\kappa(t))) e^{-r(t_f - t_0)} = p_{m,i} m_i(t_0) + \int_{t_0}^{t_f} R_i(s; \kappa(s)) e^{-rs} ds$$

If the growth has a large positive (negative) affect on future growth rates, landowners will regenerate additional (less) land in timber. Equation (A.41) also applies to stocks that will potentially shift from one type to another. In some cases, however, the landowner must decide whether or not to regenerate the old species now, although the land becomes suitable for the new species before the marginal benefits are realized. This will affect the rental value of land. If the effects of climate change occur relatively soon, the landowner may decide not to regenerate at all. However, if the effects occur far in the future, they will have little influence on the decision to regenerate

The intensity of regeneration management, as described by $m_i(t_0)$ will also be affected by future timber yields. Under the regeneration scenario, this decision is made according to

(A.42)

$$(P(t_f) - c_i^{H,i}) \frac{dV_i(t_f - t_0, m_i(t_0); \theta_i(\kappa(t)))}{dm_i(t_0)} e^{-r(t_f - t_0)} = p_{m,i}$$

For lands that are shifting from one type to another after harvest, the yield function for the future timber type is used to make this decision. In some cases in the regeneration scenario, timber that does not shift from one ecosystem type to another for several more years or decades will be harvested. The regeneration decision then must trade off the potential future benefits of another hectare of the existing type (recall that species that are regenerated will grow to maturity in the regeneration case) versus the benefits of waiting until climate has changed sufficiently to shift to the new type. The decision to establish plantations in the regeneration scenario is the same as that in the dieback scenario.