AED Econ 802
Applied Computational Economics
Homework 6
Due Monday, November 9, 2009

1. An industrial firm's profit in period t

$$\pi(q_t) = \alpha_0 + \alpha_1 q_t - 0.5 q_t^2$$

is a function of its output q_t. The firm's production process generates an environmental pollutant. Specifically, if x_t is the level of pollutant in the environment in period t, then the level of the pollutant the following period will be

$$x_{t+1} = \beta x_t + q_t$$

where $0 < \beta < 1$.

A firm operating without regard to environmental consequences produces at its profit maximizing level $q_t = \alpha_1$. Suppose that the social welfare, accounting for environmental damage, is measured by

$$\sum_{t=0}^{\infty} \delta^t \left[\pi(q_t) - c x_t\right]$$

where c is the unit social cost of suffering the pollutant and $\delta < 1$ is the social discount factor.

(a) Formulate the social planner's dynamic optimization problem. Specifically, formulate the Bellman equation, clearly identifying the state and action variables, the state and action spaces, and the reward and transition functions.

(b) Assuming an internal solution, derive the Euler conditions and interpret them. What does the shadow price function represent?

(c) Solve for the steady-state socially optimal production level q^* and pollution level x^* in terms of the model parameters $(\alpha_0, \alpha_1, \delta, \beta, c)$.

(d) Determine the per-unit tax on output τ that will induce the firm to produce at the steady-state socially optimal production level q^*.

2. Consider the problem of harvesting a renewable resource over an infinite time horizon. For year t, let s_t denote the resource stock at the beginning of the year, let x_t denote the amount of the resource harvested, let $p_t = p(x_t) = \alpha_0 - \alpha_1 x_t$ denote the market clearing price, and let $c_t = c(s_t) = \beta_0 + \beta_1 s_t$ denote the unit cost of harvest. Assume an annual interest rate r and a stock growth dynamic $s_{t+1} = s_t + \gamma(\bar{s} - s_t) - x_t$ where $\bar{s}$ is the no-harvest steady-state stock level.

   (a) Formulate the social planner's problem of maximizing the discounted sum of net social surplus over time. Specifically, formulate the Bellman equation, clearly identifying the state and action variables, the state and action spaces, and the reward and transition functions.

   (b) Formulate the monopolist's problem of maximizing the discounted sum of profits over time. Specifically, formulate the Bellman equation, clearly identifying the state and action variables, the state and action spaces, and the reward and transition functions.

   (c) Solve for the steady-state harvest and stock levels, x^* and s^*, for both the social planner and the monopolist. Who maintains the larger resource stock in steady-state?

   (d) How do the steady-state equilibrium stock levels change if demand rises (i.e., if α_0 rises)? How do they change if the harvest cost rises (i.e., if β_0 rises)?

3. At time t, a firm earns net revenue

$$\pi_t = py_t - rk_t - \tau_t k_t - c_t$$

   where p is the market price, y_t is output, r is the capital rental rate, k_t is capital at the beginning of the period, c_t is the cost of adjusting capital, and τ_t is tax paid per unit of capital. The firm's production function, adjustment costs, and tax rate are given by

$$\begin{aligned} y_t &= \alpha k_t, \\ c_t &= 0.5\beta(k_{t+1} - k_t)^2, \\ \tau_t &= \tau + 0.5\gamma k_t. \end{aligned}$$

Assume that the unit output price p and the unit capital rental rate r are both exogenously fixed and known; also assume that the parameters $\alpha > 0$, $\beta > 0$, $\gamma > 0$, and $\tau > 0$ are given.

(a) Formulate the firm's problem of maximizing the discounted sum of profits over time. Specifically, formulate the Bellman equation, clearly identifying the state and action variables, the state and action spaces, and the reward and transition functions.

(b) Assuming an internal solution, derive the Euler conditions and interpret them. What does the shadow price function represent?

(c) What effect does an increase in the base tax rate τ have on output in the long run.

(d) What effect does an increase in the discount factor δ have on output in the long run.