AED Econ 802
Applied Computational Economics
Homework 5
Due Monday, November 2, 2009

1. Consider an oligopolistic market consisting of m identical firms, each of which takes its competitors' output *decision rules* as fixed when determining its output level. That is, the typical firm assumes that its competitors' will behave as it does, so that the marginal impact of its output decision q_i on market price p satisfies

$$\frac{dp}{dq_i} = \frac{1}{d'(p)} \sum_{j=1}^{n} \frac{dq_j}{dq_i} = \frac{1}{d'(p)} \left(1 + (m-1)s'(p) \frac{dp}{dq_i} \right),$$

where $s(\cdot)$ is the representative firm's effective supply curve. Under this assumption, the representative firm's first-order profit maximization condition is

$$\frac{d\pi}{dq_i} = p + \frac{q_i}{d'(p) - (m-1)s'(p)} - k(q_i) = 0,$$

where $k(\cdot)$ is the representative firm's marginal cost function. Write a Matlab script that solves for the representative firm's effective supply curve for an arbitrary number of firms m assuming that $d(p) = p^{-\eta}$ and $k(q) = \alpha\sqrt{q} + q^2$, where $\alpha = 1$ and $\eta = 3.5$. For prices ranging between 0.4 and 0.8:

   (a) plot the approximation residual for the representative firm's effective supply function, and

   (b) on one figure, plot the demand curve and the *industry* effective supply curves $ms(\cdot)$ for $m = 10$, $m = 50$, and $m = 10$.

2. Consider the potato market model presented in Problem Set 2, Problem 1. Assume now that supply of potatoes s at the beginning of the first marketing period is the product of the acreage planted a and a random yield y that is unknown at planting time. Also assume that acreage planted is a function $a = 5 + 0.2Ep_1$ of the period 1 price expected at planting time and that log yield, conditional on the information known at planting time, is normally distributed with mean zero and standard deviation 0.2.

(a) Using the routines that you developed for Problem Set 2, construct a Chebychev polynomial approximant for the function $p_1 = f(s)$ that gives the period one price p_1 as a function the supply s available at the beginning of period 1, for supplies in the interval $[5, 20]$.

(b) Using the constructed approximant and a 5 point Gaussian quadrature scheme for log yield, solve the fixed-point problem

$$a = 5 + 0.2E_y f(ay)$$

for the rational expectations equilibrium acreage planted, employing the univariate rootfinding problem of your choice. Caution: function iteration will likely fail.

3. Use the collocation method to solve the following boundary value problem for $x : [0, 1] \mapsto \Re$:

$$(1 + t^2) \cdot x(t) - x''(t) = t^2, \quad t \in [0, 1]$$

$$x(0) = x(1) = 0.$$

Using Chebychev approximation, how many basis functions are required to ensure that the absolute value of the residual does not exceed 10^{-8}? How many orders of magnitude of accuracy are lost if you use cubic spline interpolation and the same number of basis functions?