AED Econ 802
Applied Computational Economics
Homework 4
Due Monday, October 26, 2009

1. Consider the progression of an influenza epidemic. At each point in time t, let $I(t)$ denote the percentage of individuals who are currently infected, let $R(t)$ denote the percentage of individuals who have been infected but have recovered, and let $S(t)$ denote the percentage of individuals who have not yet been infected and thus remain susceptible to infection. Population dynamics are governed by the ordinary differential equation

$$\begin{aligned} I' &= aSI - bI \\ R' &= bI \\ S' &= -aSI, \end{aligned}$$

where $a > 0$ is the contagion rate and $b > 0$ is the recovery rate.

   (a) On one figure, plot $I(t)$, $R(t)$, and $S(t)$ from $t = 0$ to $t = 40$, assuming that $a = 0.02$, $b = 0.2$, $S(0) = 99$, $I(0) = 1$, and $R(0) = 0$.

   (b) On one figure, plot $I(t)$, $R(t)$, and $S(t)$ from $t = 0$ to $t = 400$, assuming that $a = 0.001$, $b = 0.1$, $S(0) = 98$, $I(0) = 2$, and $R(0) = 0$.

2. Consider a coupled economic-ecological system in which the rate of population growth initially increases with population due to agglomeration effects and then declines with population due to congestion effects, and, in addition, is dampened by the incidence of ambient pollution. Specifically, at time t, the population $N(t)$ and ambient pollution $P(t)$ change at rates

$$\begin{aligned} N' &= \alpha_0 + \alpha_1 N - \alpha_2 N^2 - \beta P \\ P' &= \gamma N - \rho(P - \bar{P}). \end{aligned}$$

   Solve for the paths taken by population and pollution through from time $t = 0$ through time $t = 40$, starting from $N(0) = 2$ and $P(0) = 5$, assuming that $\alpha_0 = 0.2$, $\alpha_1 = 0.4$, $\alpha_2 = 0.03$, $\beta = 0.3$, $\gamma = 0.8$,

$\rho = 0.9$, and $\bar{p} = 1.0$. On one figure, with N ranging from 0 to 5 on the horizontal axis and P ranging from 0 to 5 on the vertical axis, plot

(a) the population nullcline, along which population is unchanging;

(b) the pollution nullcline, along which pollution is unchanging; and

(c) the path traversed by population and pollution.