AED Econ 802
Applied Computational Economics
Homework 2
Due Monday, October 12, 2009

1. Potatoes are storable intraseasonaly, but not interseasonaly, implying that the harvest is entirely consumed over the subsequent twelve months, $i = 1, 2, 3, \ldots, 12$. Denoting initial supply by s and consumption in period i by c_i, material balance requires that:

$$s = \sum_{i=1}^{12} c_i.$$

Competition among storers possessing perfect foresight eliminate arbitrage opportunities; thus,

$$p_i + \kappa = \delta p_{i+1}, \qquad i = 1, 2, 3, \ldots, 11,$$

where p_i is equilibrium price in period i, $\kappa = 0.02$ is per-period unit cost of storage, and $\delta = 0.99$ is per-period discount factor. Demand, assumed the same across periods, is given by

$$p_i = c_i^{-5}.$$

Plot on a single figure the equilibrium price path for $s = 10$, $s = 20$, and $s = 30$.

2. Consider a simple endowment economy with three agents and two goods. Agent i is initially endowed with e_{ij} units of good j and maximizes utility

$$U_i(x) = \sum_{j=1}^{2} a_{ij}(v_{ij} + 1)^{-1} x_{ij}^{v_{ij}+1},$$

subject to the budget constraint

$$\sum_{j=1}^{2} p_j x_{ij} = \sum_{j=1}^{2} p_j e_{ij}.$$

Here, x_{ij} is the amount of good j consumed by agent i, p_j is the market price of good j, and $a_{ij} > 0$ and $v_{ij} < 0$ are preference parameters.

A competitive general equilibrium for the endowment economy is a pair of relative prices, p_1 and p_2, normalized to sum to one, such that all the goods markets clear if each agent maximizes utility subject to his budget constraints.

Compute the competitive general equilibrium for the following parameters:

(i,j)	a_{ij}	v_{ij}	e_{ij}
(1,1)	2.0	-2.0	2.0
(1,2)	1.5	-0.5	3.0
(2,1)	1.5	-1.5	1.0
(2,2)	2.0	-0.5	2.0
(3,1)	1.5	-0.5	4.0
(3,2)	2.0	-1.5	0.0

3. The Black-Scholes option pricing formula expresses the price C of a European call option on a stock in terms of the price S of the underlying stock, the strike price K, the time in years τ until the expiration of the option, the continuously compounded risk-free interest rate r, and the annualized volatility σ of the price of the underlying stock:

$$C = S\Phi(d) - Ke^{-r\tau}\Phi(d - \sigma\sqrt{\tau})$$

where

$$d = \frac{\ln(S/K) + (r + 0.5\sigma^2)\tau}{\sigma\sqrt{\tau}},$$

and Φ is the standard normal cumulative distribution function.

All of the inputs to the Black-Scholes formula are readily observable, except the volatility σ of underlying stock price. Compute the "implied volatility" of the underlying stock price if a stock call option expiring in one year with strike price $K = 1.1$ is currently trading a price $C = 0.0728$, the underlying stock is trading at a price $S = 1$, and the risk-free interest rate is $r = 0.08$.

Note: The Matlab Statistics Toolbox functions `normcdf` and `normpdf` may be used to evaluate normal cumulative and probability density functions.